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## ENERGY SPECTRA OF ANTI-NUCLEONS IN FINITE NUCLEI

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The quantum vacuum in a many-body system of finite nuclei has been investigated within the relativistic Hartree approach which describes the bound states of nucleons and anti-nucleons consistently. The contributions of the Dirac sea to the source terms of the meson-field equations are taken into account up to the one-nucleon loop and one-meson loop. The tensor couplings for the  $\omega$ - and  $\rho$ -meson are included in the model. The overall nucleon spectra of shell-model states are in agreement with the data. The calculated anti-nucleon spectra in the vacuum differ about 20 – 30 MeV with and without the tensor-coupling effects.

It is quite interesting to study the structure of quantum vacuum in a many-body system, e.g., in a finite nucleus where the Fermi sea is filled with the valence nucleons while the Dirac sea is full of the virtual nucleon–anti-nucleon pairs <sup>1,2</sup>. The shell-model states have been theoretically and experimentally well established <sup>3</sup>. It is the aim of our work to investigate the bound states of anti-nucleons in the Dirac Sea. The observation of anti-nucleon bound states is a verification for the application of the relativistic quantum field theory to a many-body system <sup>4</sup>. Since the bound states of nucleons are subject to the cancellation of scalar and vector potentials  $S + V$  ( $V$  is positive,  $S$  is negative) while the bound states of anti-nucleons are sensitive to the sum of them  $S - V$ , consistent studies of both the nucleon and the anti-nucleon bound states can determine the individual  $S$  and  $V$ . In addition, the knowledge of potential depth for anti-nucleons in the medium is a prerequisite for the study of anti-matter and anti-nuclei in relativistic heavy-ion collisions.

We have developed a relativistic Hartree approach which describes the bound states of nucleons and anti-nucleons in a unified framework <sup>5,6</sup>. In finite nuclei the Dirac equation of nucleons is written as

$$i \frac{\partial}{\partial t} \psi(\mathbf{x}, t) = \left[ -i \boldsymbol{\alpha} \cdot \boldsymbol{\nabla} + \beta (M_N - g_\sigma \sigma(\mathbf{x})) + g_\omega \omega_0(\mathbf{x}) - \frac{f_\omega}{2M_N} i \boldsymbol{\gamma} \cdot (\boldsymbol{\nabla} \omega_0(\mathbf{x})) \right]$$

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$$+ \frac{1}{2}g_\rho\tau_0 R_{0,0}(\mathbf{x}) - \frac{f_\rho}{4M_N}i\tau_0\boldsymbol{\gamma} \cdot (\boldsymbol{\nabla} R_{0,0}(\mathbf{x})) + \frac{1}{2}e(1+\tau_0)A_0(\mathbf{x}) \Big] \psi(\mathbf{x}, t) \quad (1)$$

The field operator can be expanded according to nucleons and anti-nucleons and reads as

$$\psi(\mathbf{x}, t) = \sum_{\alpha} \left[ b_{\alpha} \psi_{\alpha}(\mathbf{x}) e^{-iE_{\alpha}t} + d_{\alpha}^{\dagger} \psi_{\alpha}^a(\mathbf{x}) e^{i\bar{E}_{\alpha}t} \right]. \quad (2)$$

Here the label  $\alpha$  denotes the full set of single-particle quantum numbers. The wave functions of nucleons and anti-nucleons can be specified as <sup>7,8</sup>

$$\psi_{\alpha}(\mathbf{x}) = \begin{pmatrix} i \frac{G_{\alpha}(r)}{r} \Omega_{jlm}(\frac{\mathbf{r}}{r}) \\ \frac{F_{\alpha}(r)}{r} \frac{\boldsymbol{\sigma} \cdot \mathbf{r}}{r} \Omega_{jlm}(\frac{\mathbf{r}}{r}) \end{pmatrix}, \quad (3)$$

$$\psi_{\alpha}^a(\mathbf{x}) = \begin{pmatrix} -\frac{\bar{F}_{\alpha}(r)}{r} \frac{\boldsymbol{\sigma} \cdot \mathbf{r}}{r} \Omega_{jlm}(\frac{\mathbf{r}}{r}) \\ i \frac{\bar{G}_{\alpha}(r)}{r} \Omega_{jlm}(\frac{\mathbf{r}}{r}) \end{pmatrix}, \quad (4)$$

where  $\Omega_{jlm}$  are the spherical spinors. Inserting Eq. (2) into Eq. (1) we obtain the following equations for the upper component of the nucleon's wave function

$$E_{\alpha} G_{\alpha}(r) = \left[ -\frac{d}{dr} + W(r) \right] M_{eff}^{-1} \left[ \frac{d}{dr} + W(r) \right] G_{\alpha}(r) + U_{eff} G_{\alpha}(r), \quad (5)$$

and the lower component of the anti-nucleon's wave function

$$\bar{E}_{\alpha} \bar{G}_{\alpha}(r) = \left[ -\frac{d}{dr} + \bar{W}(r) \right] \bar{M}_{eff}^{-1} \left[ \frac{d}{dr} + \bar{W}(r) \right] \bar{G}_{\alpha}(r) + \bar{U}_{eff} \bar{G}_{\alpha}(r). \quad (6)$$

Other components can be obtained through the following relations

$$F_{\alpha}(r) = M_{eff}^{-1} \left[ \frac{d}{dr} + W(r) \right] G_{\alpha}(r), \quad (7)$$

$$\bar{F}_{\alpha}(r) = \bar{M}_{eff}^{-1} \left[ \frac{d}{dr} + \bar{W}(r) \right] \bar{G}_{\alpha}(r). \quad (8)$$

The Schrödinger-equivalent effective masses and potentials are defined as follows: for the nucleon

$$M_{eff} = E_{\alpha} + M_N - g_{\sigma}\sigma(r) - g_{\omega}\omega_0(r) - \frac{1}{2}g_{\rho}\tau_{0\alpha}R_{0,0}(r) - \frac{1}{2}e(1+\tau_{0\alpha})A_0(r), \quad (9)$$

$$U_{eff} = M_N - g_{\sigma}\sigma(r) + g_{\omega}\omega_0(r) + \frac{1}{2}g_{\rho}\tau_{0\alpha}R_{0,0}(r) + \frac{1}{2}e(1+\tau_{0\alpha})A_0(r), \quad (10)$$

$$W(r) = \frac{\kappa_{\alpha}}{r} - \frac{f_{\omega}}{2M_N}(\partial_r\omega_0(r)) - \frac{f_{\rho}}{4M_N}\tau_{0\alpha}(\partial_r R_{0,0}(r)), \quad (11)$$

for the anti-nucleon

$$\bar{M}_{eff} = \bar{E}_{\alpha} + M_N - g_{\sigma}\sigma(r) + g_{\omega}\omega_0(r) - \frac{1}{2}g_{\rho}\tau_{0\alpha}R_{0,0}(r) + \frac{1}{2}e(1+\tau_{0\alpha})A_0(r), \quad (12)$$

$$\bar{U}_{eff} = M_N - g_{\sigma}\sigma(r) - g_{\omega}\omega_0(r) + \frac{1}{2}g_{\rho}\tau_{0\alpha}R_{0,0}(r) - \frac{1}{2}e(1+\tau_{0\alpha})A_0(r), \quad (13)$$

$$\bar{W}(r) = \frac{\kappa_{\alpha}}{r} + \frac{f_{\omega}}{2M_N}(\partial_r\omega_0(r)) - \frac{f_{\rho}}{4M_N}\tau_{0\alpha}(\partial_r R_{0,0}(r)). \quad (14)$$

One can see that the difference between the equations of nucleons and anti-nucleons relies only on the definition of the effective masses and potentials. The main ingredients are meson fields which can be obtained through solving the Laplace equations of mesons. The source terms are various densities containing the contributions both from the valence nucleons and the Dirac sea. They are evaluated by means of the derivative expansion technique<sup>9</sup>. The energy spectra of the nucleon and the anti-nucleon are computed by means of the equations

$$E_\alpha = \int_0^\infty dr \{ G_\alpha(r) \left[ -\frac{d}{dr} + W(r) \right] F_\alpha(r) + F_\alpha(r) \left[ \frac{d}{dr} + W(r) \right] G_\alpha(r) + G_\alpha(r) U_{eff} G_\alpha(r) - F_\alpha(r) [M_{eff} - E_\alpha] F_\alpha(r) \}, \quad (15)$$

$$\bar{E}_\alpha = \int_0^\infty dr \{ \bar{G}_\alpha(r) \left[ -\frac{d}{dr} + \bar{W}(r) \right] \bar{F}_\alpha(r) + \bar{F}_\alpha(r) \left[ \frac{d}{dr} + \bar{W}(r) \right] \bar{G}_\alpha(r) + \bar{G}_\alpha(r) \bar{U}_{eff} \bar{G}_\alpha(r) - \bar{F}_\alpha(r) [\bar{M}_{eff} - \bar{E}_\alpha] \bar{F}_\alpha(r) \}. \quad (16)$$

The parameters of the model are fitted to the properties of spherical nuclei<sup>5,6</sup>. The major result is that a large effective nucleon mass  $m^*/M_N \approx 0.78$  is obtained. In Table 1 we present the single-particle energies of protons (neutrons) and anti-protons (anti-neutrons) in <sup>16</sup>O, <sup>40</sup>Ca and <sup>208</sup>Pb. The binding energies per nucleon and the *rms* charge radii are given too. The experimental data are taken from Ref. <sup>10</sup>. It can be found that the relativistic Hartree approach taking into account the vacuum effects can reproduce the observed binding energies, *rms* charge radii and particle spectra quite well. Because of the large effective nucleon mass, the spin-orbit splitting on the *1p* levels is rather small in the RHA1 model. The situation has been ameliorated conspicuously in the RHAT model incorporating the tensor couplings for the  $\omega$ - and  $\rho$ -meson, while a large  $m^*$  stays unchanged. On the other hand, the anti-particle energies calculated with the RHAT set of parameters are 20 – 30 MeV larger than that reckoned with the RHA1 set.

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Table 1. The single-particle energies of both protons (neutrons) and anti-protons (anti-neutrons) as well as the binding energies per nucleon and the *rms* charge radii in  $^{16}\text{O}$ ,  $^{40}\text{Ca}$  and  $^{208}\text{Pb}$ .

|                        | RHA1   | RHAT   | Expt.       |                        | RHA1   | RHAT   | Expt. |
|------------------------|--------|--------|-------------|------------------------|--------|--------|-------|
| $^{16}\text{O}$        |        |        |             |                        |        |        |       |
| $E/A$ (MeV)            | 8.00   | 7.94   | 7.98        |                        |        |        |       |
| $r_{ch}$ (fm)          | 2.66   | 2.64   | 2.74        |                        |        |        |       |
| PROTONS                |        |        |             | NEUTRONS               |        |        |       |
| $1s_{1/2}$ (MeV)       | 30.68  | 31.63  | $40 \pm 8$  | $1s_{1/2}$ (MeV)       | 34.71  | 35.78  | 45.7  |
| $1p_{3/2}$ (MeV)       | 15.23  | 16.18  | 18.4        | $1p_{3/2}$ (MeV)       | 19.04  | 20.18  | 21.8  |
| $1p_{1/2}$ (MeV)       | 13.24  | 12.22  | 12.1        | $1p_{1/2}$ (MeV)       | 17.05  | 15.75  | 15.7  |
| ANTI-PRO.              |        |        |             | ANTI-NEU.              |        |        |       |
| $1\bar{s}_{1/2}$ (MeV) | 299.42 | 328.55 |             | $1\bar{s}_{1/2}$ (MeV) | 293.23 | 322.47 |       |
| $1\bar{p}_{3/2}$ (MeV) | 258.40 | 283.44 |             | $1\bar{p}_{3/2}$ (MeV) | 252.48 | 277.94 |       |
| $1\bar{p}_{1/2}$ (MeV) | 258.93 | 285.87 |             | $1\bar{p}_{1/2}$ (MeV) | 252.97 | 279.22 |       |
| $^{40}\text{Ca}$       |        |        |             |                        |        |        |       |
| $E/A$ (MeV)            | 8.73   | 8.62   | 8.55        |                        |        |        |       |
| $r_{ch}$ (fm)          | 3.42   | 3.41   | 3.45        |                        |        |        |       |
| PROTONS                |        |        |             | NEUTRONS               |        |        |       |
| $1s_{1/2}$ (MeV)       | 36.58  | 37.01  | $50 \pm 11$ | $1s_{1/2}$ (MeV)       | 44.48  | 44.98  |       |
| $1p_{3/2}$ (MeV)       | 25.32  | 25.95  |             | $1p_{3/2}$ (MeV)       | 32.98  | 33.83  |       |
| $1p_{1/2}$ (MeV)       | 24.03  | 23.63  | $34 \pm 6$  | $1p_{1/2}$ (MeV)       | 31.71  | 30.99  |       |
| ANTI-PRO.              |        |        |             | ANTI-NEU.              |        |        |       |
| $1\bar{s}_{1/2}$ (MeV) | 339.83 | 367.90 |             | $1\bar{s}_{1/2}$ (MeV) | 327.96 | 355.70 |       |
| $1\bar{p}_{3/2}$ (MeV) | 309.24 | 332.10 |             | $1\bar{p}_{3/2}$ (MeV) | 298.04 | 321.07 |       |
| $1\bar{p}_{1/2}$ (MeV) | 309.52 | 333.37 |             | $1\bar{p}_{1/2}$ (MeV) | 298.26 | 322.15 |       |
| $^{208}\text{Pb}$      |        |        |             |                        |        |        |       |
| $E/A$ (MeV)            | 7.93   | 7.88   | 7.87        |                        |        |        |       |
| $r_{ch}$ (fm)          | 5.49   | 5.46   | 5.50        |                        |        |        |       |
| PROTONS                |        |        |             | NEUTRONS               |        |        |       |
| $1s_{1/2}$ (MeV)       | 40.80  | 41.74  |             | $1s_{1/2}$ (MeV)       | 47.40  | 46.70  |       |
| $1p_{3/2}$ (MeV)       | 36.45  | 37.38  |             | $1p_{3/2}$ (MeV)       | 42.66  | 42.31  |       |
| $1p_{1/2}$ (MeV)       | 36.21  | 37.18  |             | $1p_{1/2}$ (MeV)       | 42.45  | 41.64  |       |
| ANTI-PRO.              |        |        |             | ANTI-NEU.              |        |        |       |
| $1\bar{s}_{1/2}$ (MeV) | 354.18 | 377.37 |             | $1\bar{s}_{1/2}$ (MeV) | 313.18 | 334.39 |       |
| $1\bar{p}_{3/2}$ (MeV) | 344.48 | 366.95 |             | $1\bar{p}_{3/2}$ (MeV) | 304.61 | 325.41 |       |
| $1\bar{p}_{1/2}$ (MeV) | 344.52 | 367.24 |             | $1\bar{p}_{1/2}$ (MeV) | 304.61 | 325.28 |       |

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